מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

COLUMBIA UNIVERSITY

Understanding the Capacity of Information Retrieval from Long Term Memory

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Bennet Murdock (Toronto) and Mike Kahana (Upenn)

Memory retrieval

Memory retrieval – with cues

Memory retrieval – without cues

Free recall VS Recognition

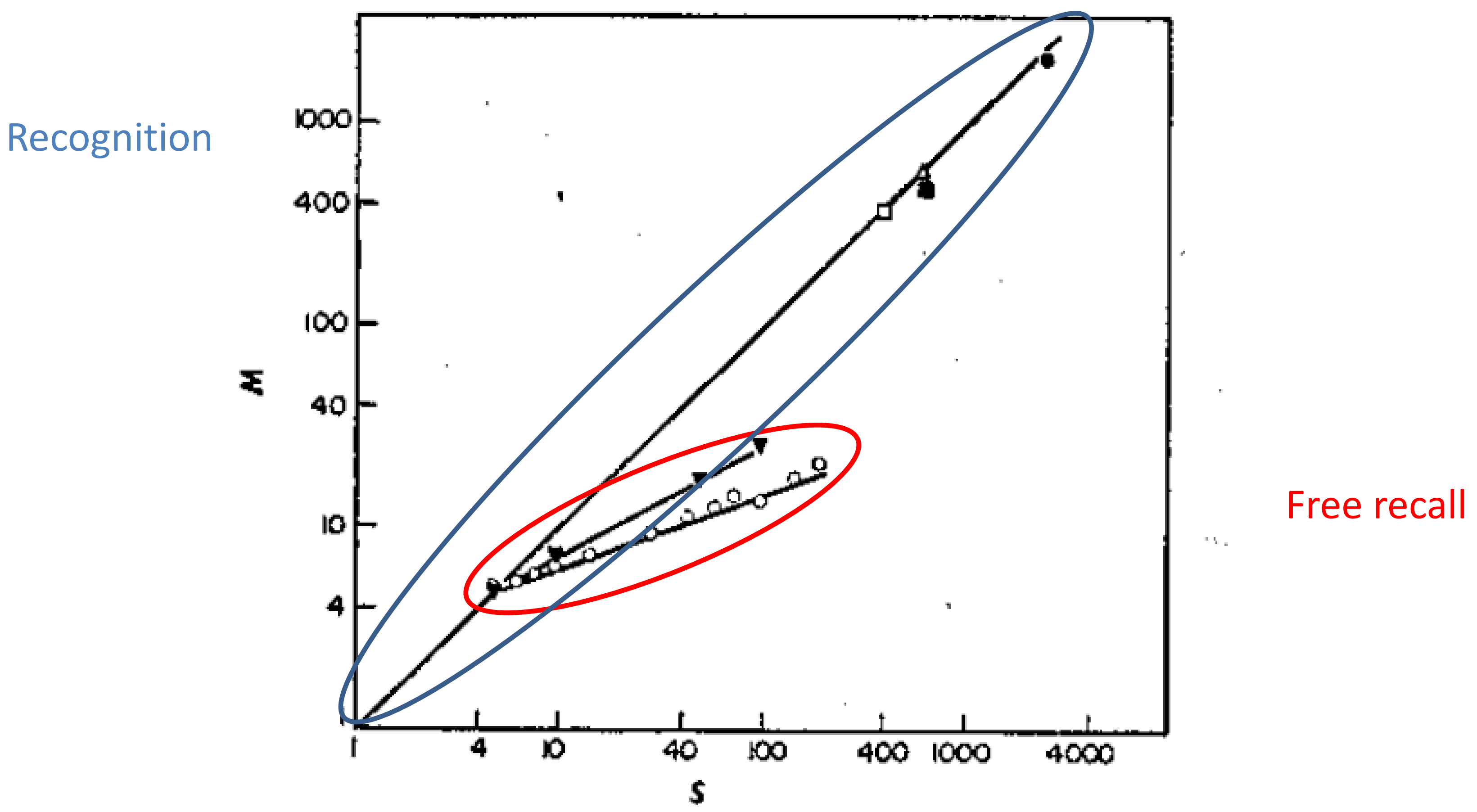


Fig: Standing (1973), *Q J Exp Psy.* Free Recall: Binet & Henri (1894), Murdock (1960) *J Exp Psy*

Journal of Experimental Psychology: Human Learning and Memory
1975, Vol. 104, No. 1, 65–70

Graphemically Cued Retrieval of Words from Long-Term Memory

D. J. Murray, *Queen's University, Kingston, Ontario, Canada*

Subjects were asked to produce in 2 min as many words as they could in which the 1st, 2nd, 3rd, 4th, or 5th letter was A or B or . . . or Z. It was found that the number of words produced was a power function of the number of words we estimated they would know in which the 1st, 2nd, 3rd, 4th, or 5th letter was A or B or . . . or Z (the vocabulary size). Also, with easy retrieval cues, high-frequency words were produced first, which was not the case for difficult retrieval cues. The relationship between word frequency and vocabulary size was also examined.

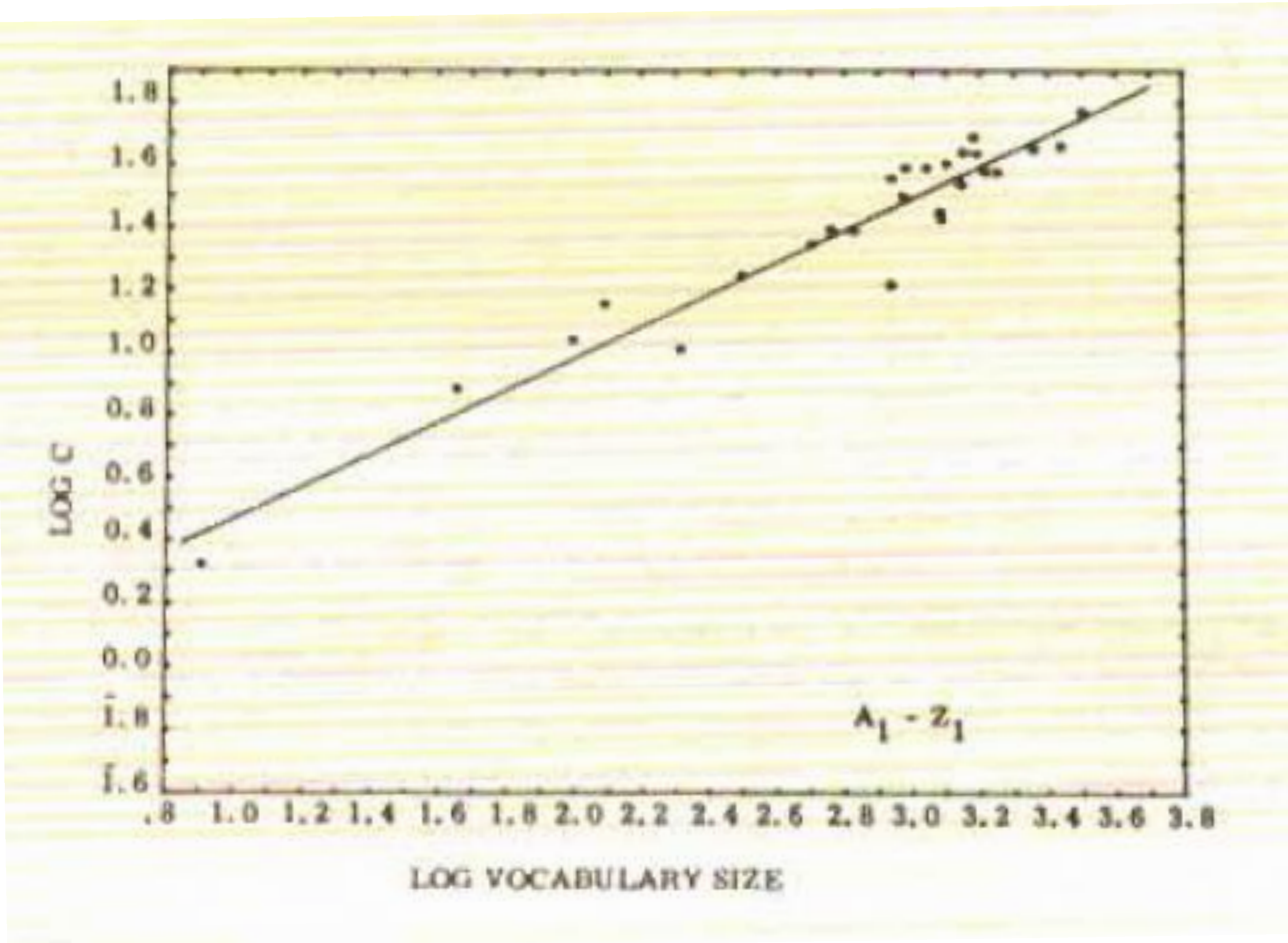
TABLE 1

RESULTS OF THE ANALYSIS OF THORNDIKE-LORGE LISTS

Letter	Cue distance 1 (a)	1 (b)	2 (a)	2 (b)	3 (a)	3 (b)	4 (a)	4 (b)	5 (a)	5 (b)
A	1,797	58.13	4,222	19.21	2,466	18.31	1,897	9.45	1,967	6.76
B	1,605	25.17	214	16.73	677	6.12	540	7.31	435	3.29
C	2,761	12.14	429	12.04	1,399	13.04	1,231	12.24	848	9.54
D	1,651	13.33	225	11.00	937	50.92	1,148	15.15	718	18.97
E	1,207	13.69	4,291	22.10	2,065	62.96	3,341	20.78	3,606	14.54
F	1,270	27.32	121	350.03	509	15.61	535	8.05	347	5.42
G	940	14.61	96	23.32	765	12.49	787	9.46	501	15.18
H	1,105	45.60	1,190	114.31	285	10.59	732	28.85	865	15.31
I	1,221	56.64	2,724	22.13	1,674	20.04	2,247	11.57	2,451	9.10
J	316	12.59	10	0.90	75	10.44	50	5.80	10	1.00
K	207	17.28	64	6.53	139	28.55	578	14.08	374	10.75
L	974	17.55	1,301	14.02	1,920	13.44	1,676	16.81	1,702	12.41
M	1,601	19.40	492	11.35	1,180	18.98	849	16.88	684	6.24
N	577	32.51	2,074	45.53	2,423	13.44	1,604	19.88	1,601	11.72
O	676	98.43	4,142	30.18	1,721	23.36	1,573	12.76	1,786	10.40
P	2,251	11.57	527	14.75	1,082	9.21	983	10.08	566	6.71
Q	124	12.50	67	8.18	79	6.16	69	5.39	25	2.80
R	1,401	11.49	2,421	17.34	3,013	19.47	1,952	16.76	2,121	13.67
S	3,188	16.56	282	76.24	1,913	25.00	1,500	17.19	1,360	7.23
T	1,416	107.82	652	41.08	1,845	30.30	2,511	20.44	1,904	12.64
U	872	9.99	2,116	15.48	1,086	20.73	1,038	13.35	897	8.65
V	512	8.96	215	23.47	514	24.56	399	11.58	194	9.49
W	875	65.53	187	23.34	318	32.66	258	20.66	219	6.74
X	7	0.86	284	11.60	124	13.67	71	2.06	35	33.49
Y	99	69.87	326	25.83	286	30.73	260	42.44	549	12.39
Z	45	2.58	16	1.69	93	5.00	96	4.93	53	5.23
Blank	0	—	9	3,164.67	110	1,802.72	722	533.14	2,880	196.50

Note. The words are from Parts 1 and 2.i. of the lists of Thorndike and Lorge (1944). Column (a) is the number of words in which the 1st, 2nd, 3rd, 4th, or 5th letters are A or B or . . . or Z. Column (b) is the mean frequency of words in which the 1st, 2nd, 3rd, 4th, or 5th letters are A or B or . . . or Z. In the latter analysis a value of 0 was given to all words in Part 2.i.

Retrieval from long-term memory – power law



$$C \approx V^{\frac{1}{2}}$$

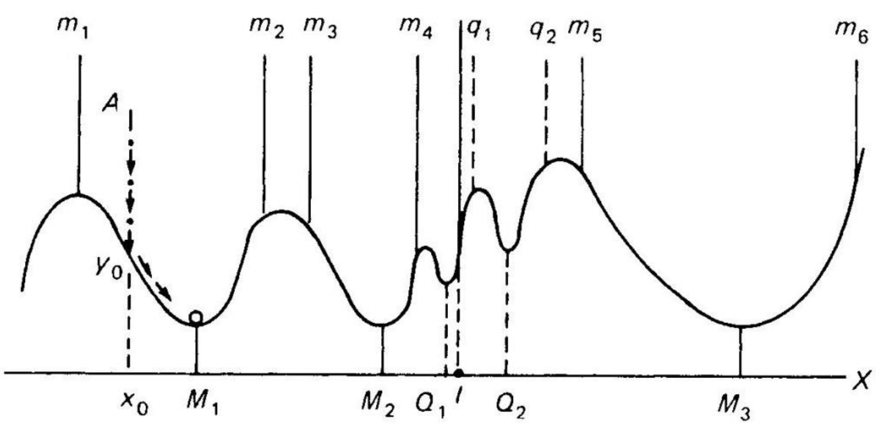
Research Questions

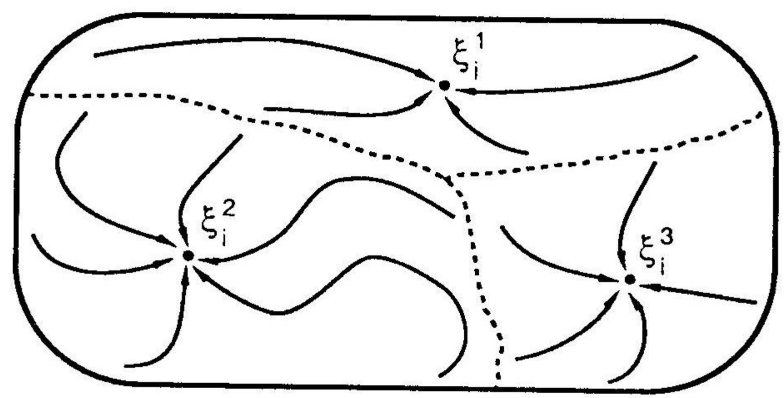
- What prevents information stored in long-term memory to be efficiently retrieved?
- Is there a parsimonious explanation for the power-law scaling of recall capacity?

Neural network models of long-term memory (Hopfield, 1982)

Memories are represented as attractors (stable states) of network dynamics.

- Attractor = internal representation (memory) of a stimulus
- Each attractor: a subset of neurons that has elevated persistent activity.
- Synaptic changes => Changes in attractor landscape = changes in memory
- Convergence to an attractor = recall of item from memory





Hopfield model with *sparse random* ***coding***

Neurons (N): $i = 1,\dots,N$

Connections (N^2): J_{ij}

Memory patterns (L): $\xi_i^\mu = 0,1 \quad \mathrm{Pr}\,ob(\xi_i^\mu = 1) = f$

$\mu = 1,\dots,L \qquad f << 1$

Storage: $\Delta J_{ij}(\mu) = (\xi_i^\mu - f)(\xi_i^\mu - f)$

(Tsodyks, Feigelman 1988)

Hopfield model with *sparse random* coding:

Storage capacity

$$P_{\max} \approx \frac{1}{2f\log(\frac{1}{f})} N$$

N: number of neurons in the network

f: average fraction of neurons in the network encoding a memory

Tsodyks & Feigelman 1988

Mathematical model

Similarities (intersections)

$$S_{\mu,\nu} = \sum_{i=1}^{N} \xi_i^{\mu} \xi_i^{\nu}$$

$$\begin{pmatrix} \times & S_{1,2} & \cdots & S_{1,L} \\ S_{2,1} & \times & \cdots & S_{2,L} \\ \vdots & & \ddots & \vdots \\ S_{L,1} & S_{L,2} & \cdots & \times \end{pmatrix}$$

Mathematical model

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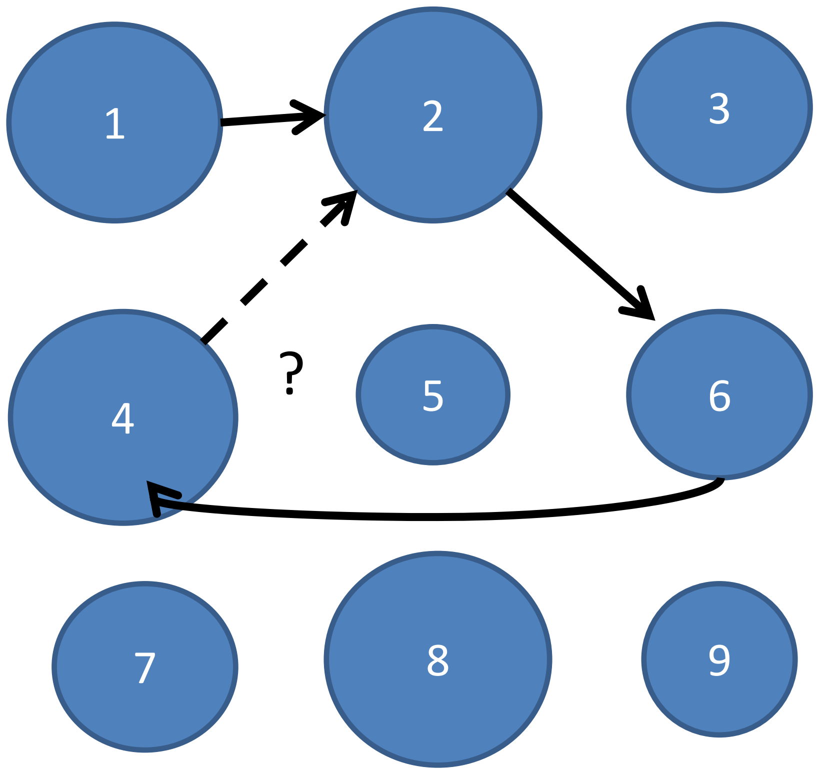
One parameter (f)

Associative retrieval: graph representation



$$1 \to 2$$

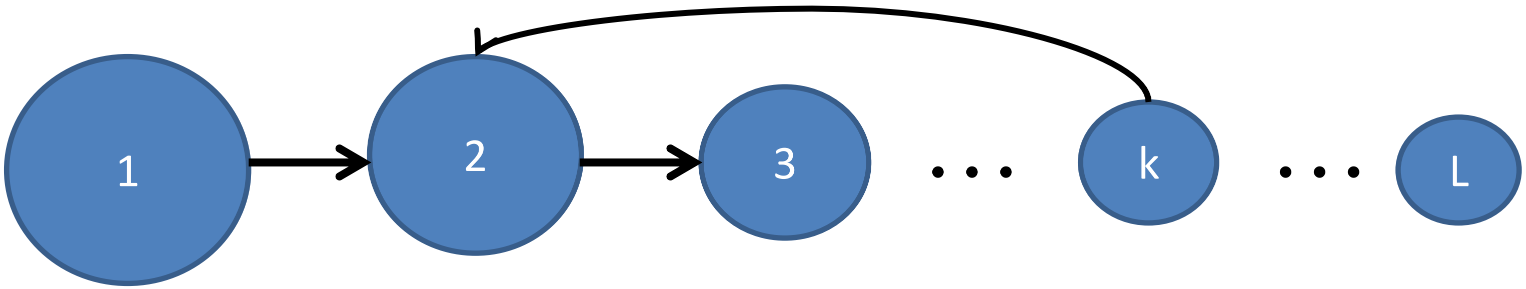
$$2 \to 6$$

$$6 \to 4$$

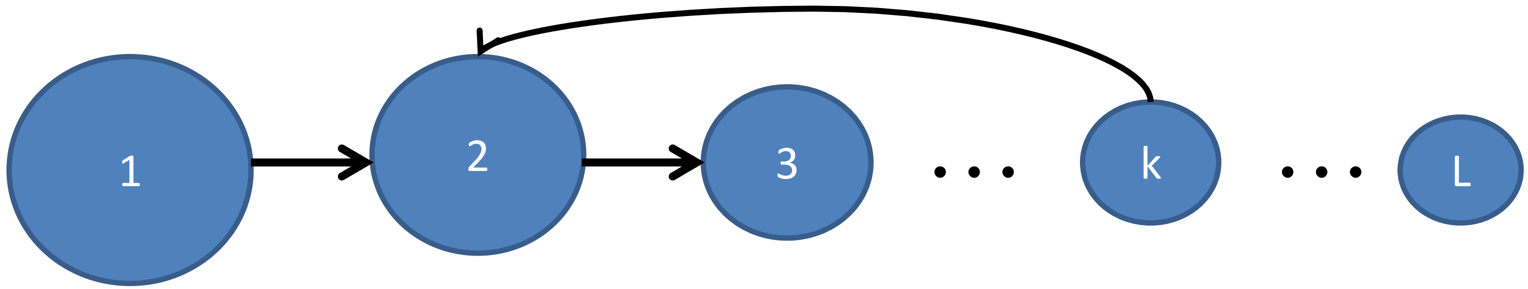
$$\ldots \to \ldots$$

Romani et al 2013

Retrieval capacity: analytical solution



Retrieval capacity: analytical solution

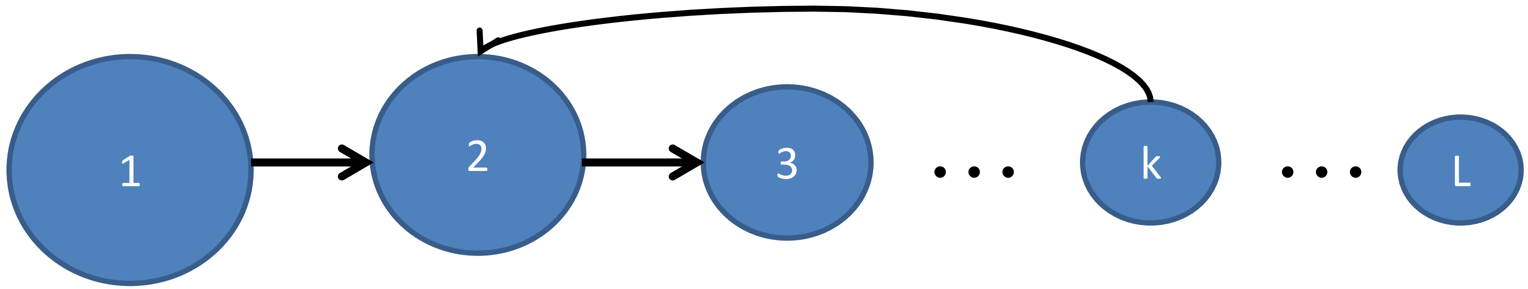


$$< Nwr > \sim L^{\alpha} \qquad \alpha = \frac{1}{2}\frac{1-f}{1+f}$$

$$\mathrm{var}(Nwr) \sim L^{2\alpha}$$

Romani et al 2013

Retrieval capacity: analytical solution



Паста+шоколадка

$$< Nwr > \sim L^{\alpha} \qquad \alpha = \frac{1}{2}\frac{1-f}{1+f}$$

$$\mathrm{var}(Nwr) \sim L^{2\alpha}$$

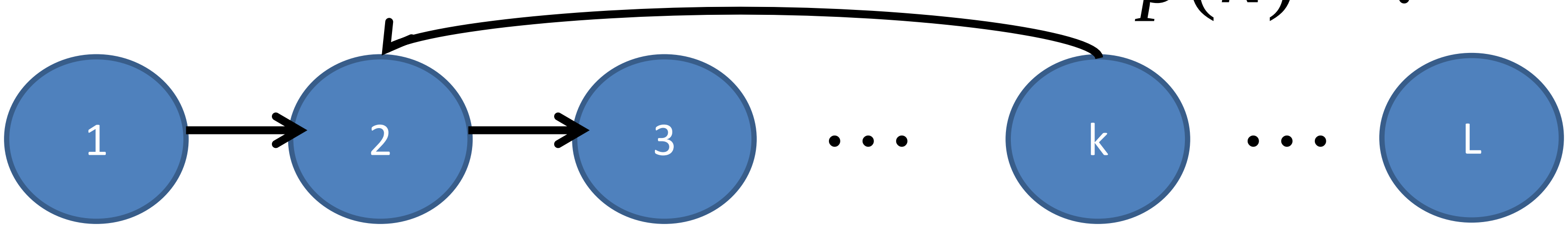
‘Naïve model’: $\mathrm{var}(Nwr) \sim L^{\alpha}$

Romani et al 2013

1. Random asymmetric matrix of similarities: exact solution of the model

$$p_0 = \frac{1}{L-1}$$

$$p(k) = ?$$



$$p(k) = (1 - \frac{1}{L-1})(1 - \frac{2}{L-1})\ldots(1 - \frac{k-2}{L-1})\frac{k-1}{L-1}$$

1. Random asymmetric matrix of similarities: exact solution of the model

$$p_0 = \frac{1}{L-1}$$

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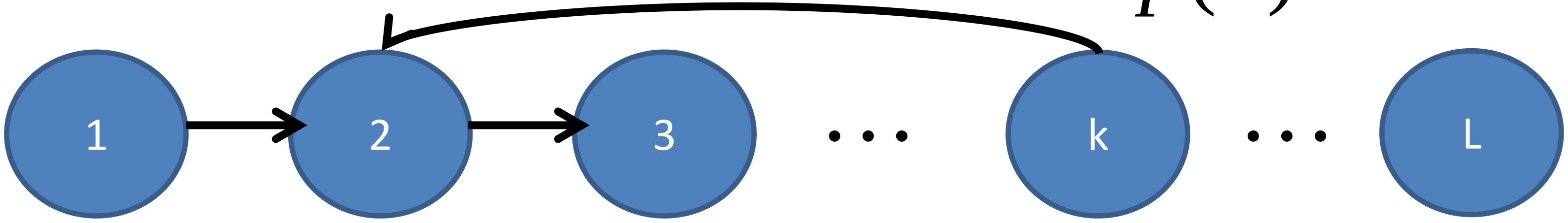
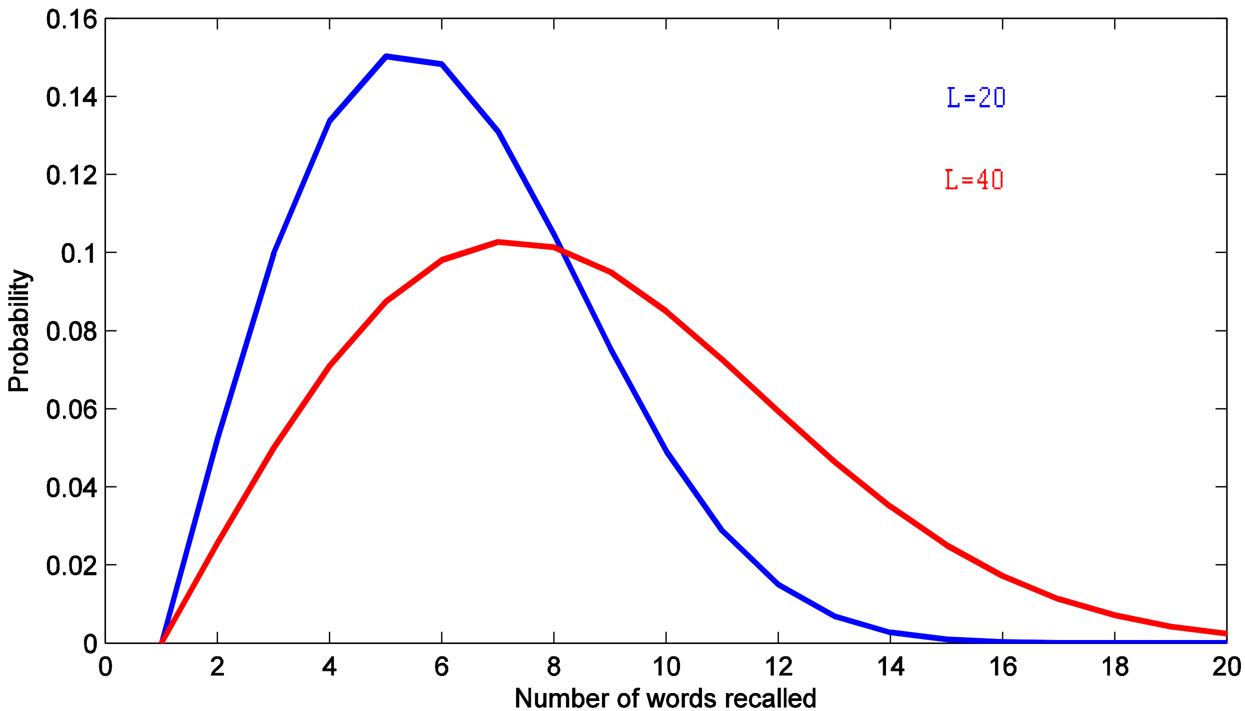
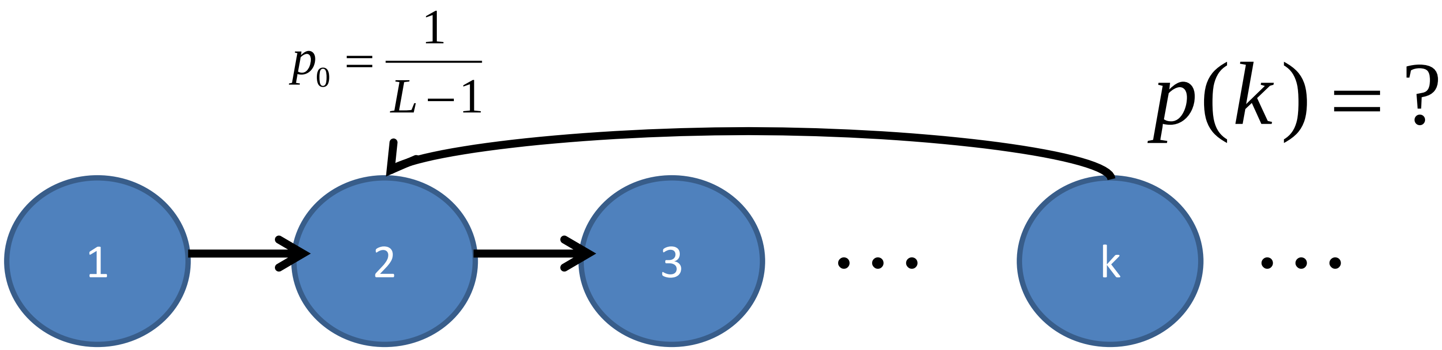




Power law scaling



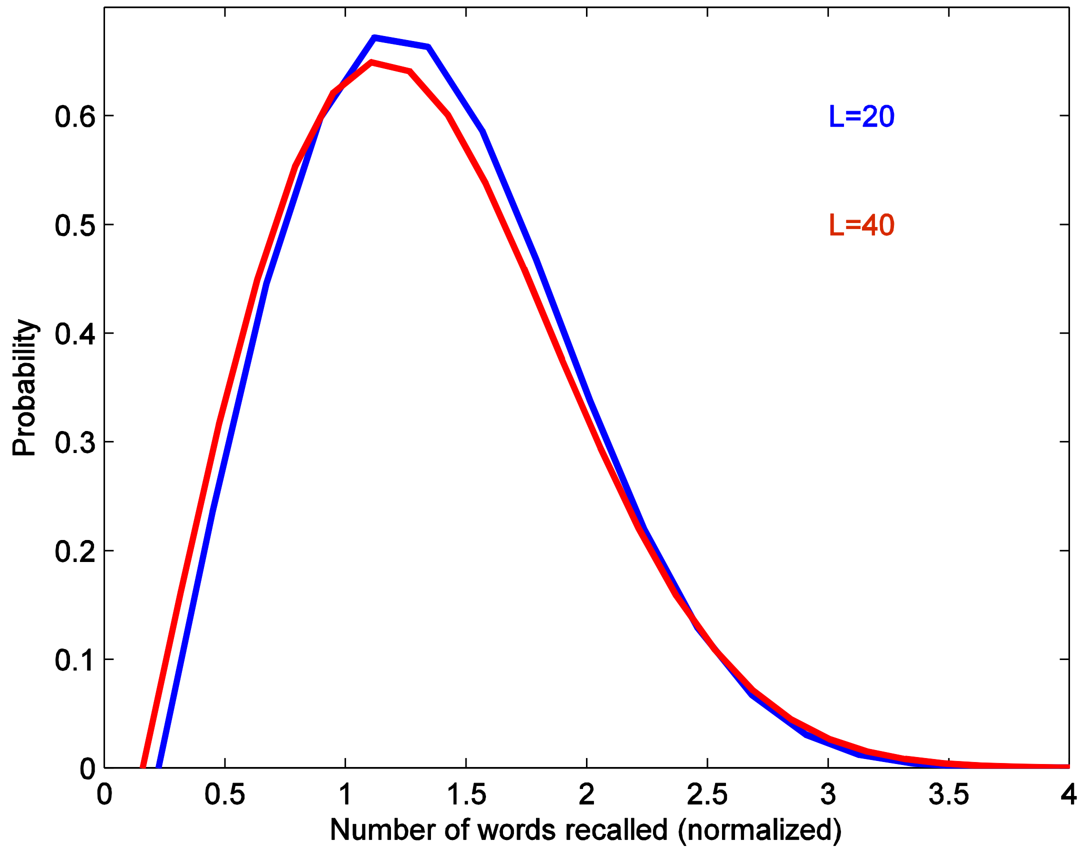
$$p(k) = (1-\frac{1}{L-1})(1-\frac{2}{L-1})\ldots(1-\frac{k-1}{L-1})\frac{k}{L-1}$$

$$k << L$$

$$\approx \frac{k}{L}\exp\left(-\frac{1}{L}\sum_{i=1}^{k} i\right) \approx \frac{k}{L}\exp\left(-\frac{k^2}{2L}\right)$$

$$x = \frac{k}{\sqrt{L}} \Rightarrow p(x) = x\exp(-\frac{x^2}{2})$$

Normalized probability distribution

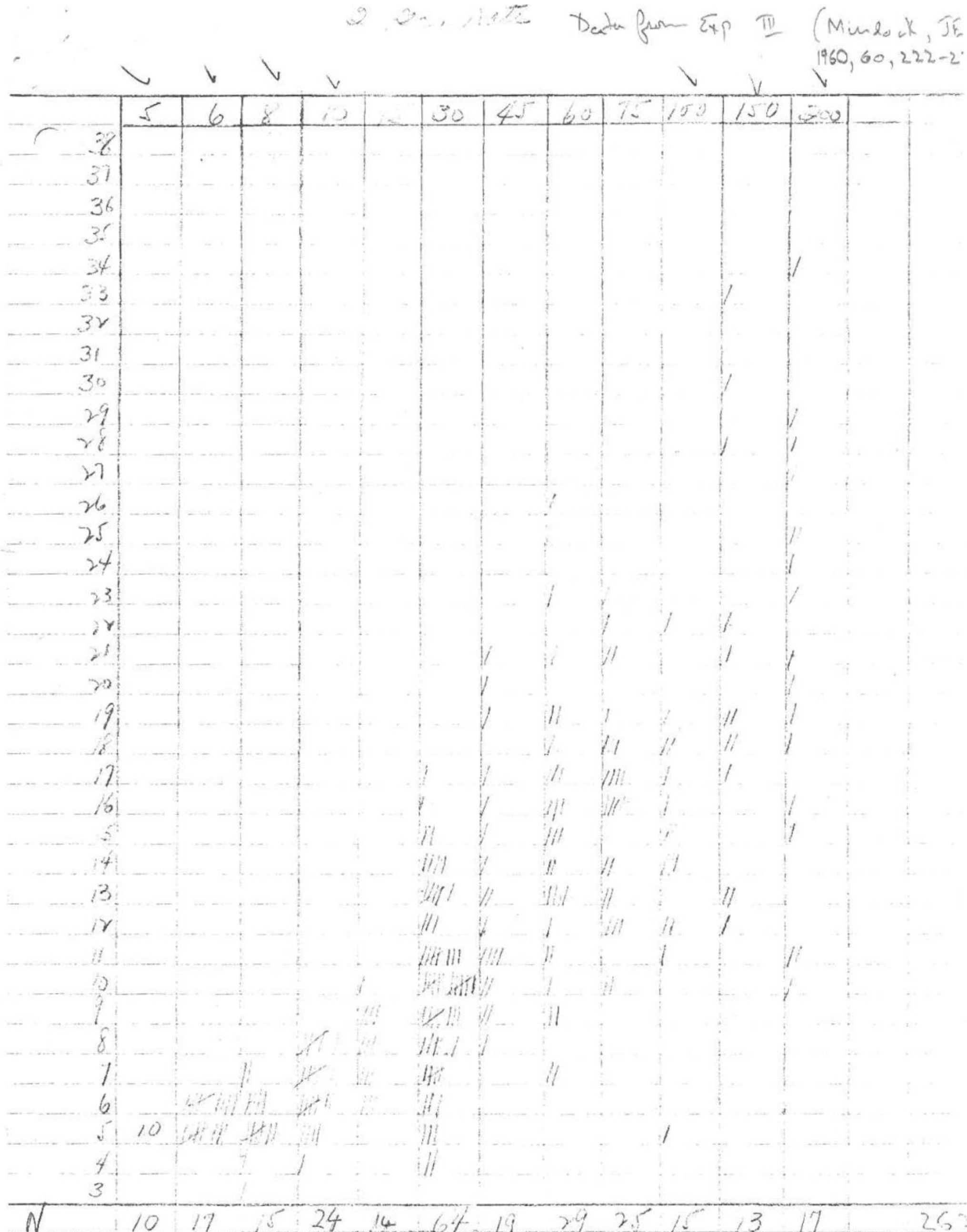


$$\langle k \rangle \approx \sqrt{\frac{\pi}{2}} \cdot L^{1/2}$$

$$Var(k) \approx (2 - \frac{\pi}{2})L$$

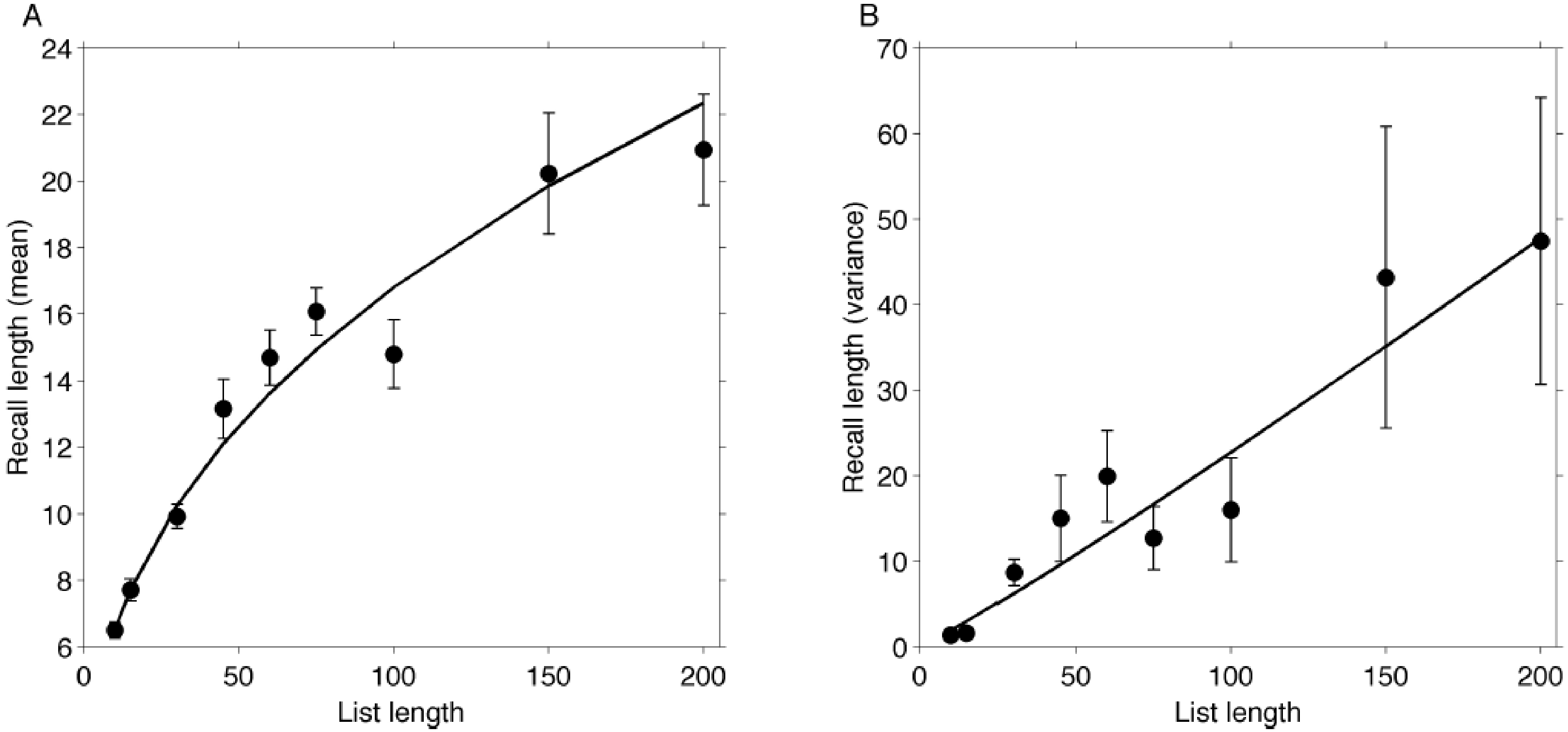
Bennet Murdock
(Toronto)

Retrieval capacity: longer lists



Courtesy of B. Murdock

Retrieval capacity: longer lists (data courtesy B. Murdock)



$$\left\langle Nwr \right\rangle \propto L^{0.41}$$

$$Var(Nwr) \propto L^{1.08}$$

Research Questions

- What prevents information stored in long-term memory to be efficiently retrieved? Answer: randomness of long-term memory representations that results in repeated recall of same items.
- Is there a parsimonious explanation for the power-law scaling of recall capacity? Answer: power-law scaling emerges from random distribution of transitions between different items.

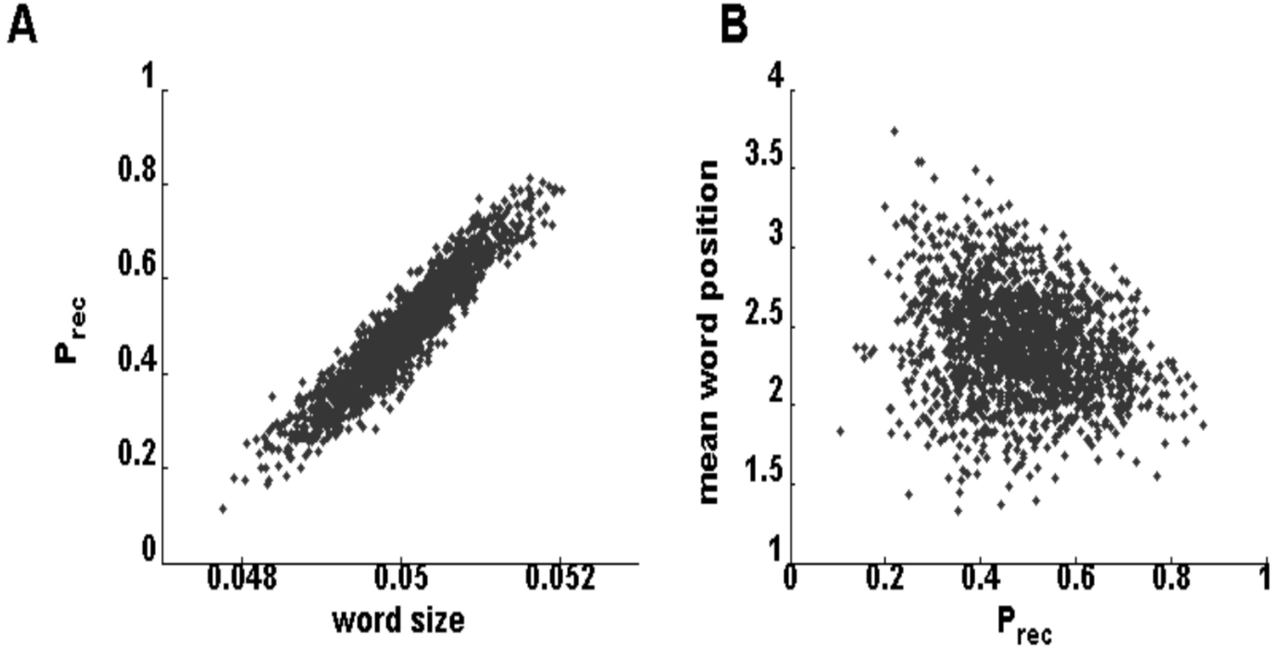
Free recall data set
(Mike Kahana, Upenn)

170 subjects

112 trials/6 sessions per subject

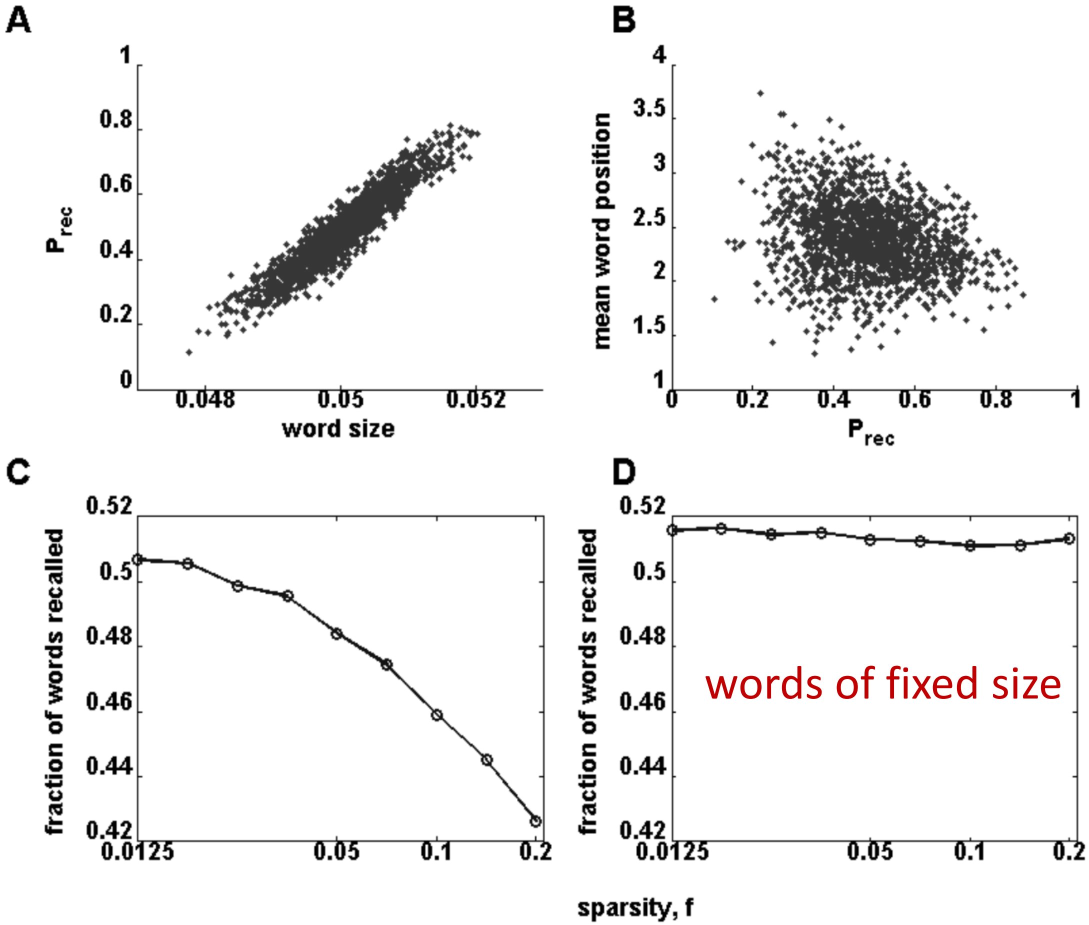
L=16 words per list

‘Easy’ vs ‘difficult’ words



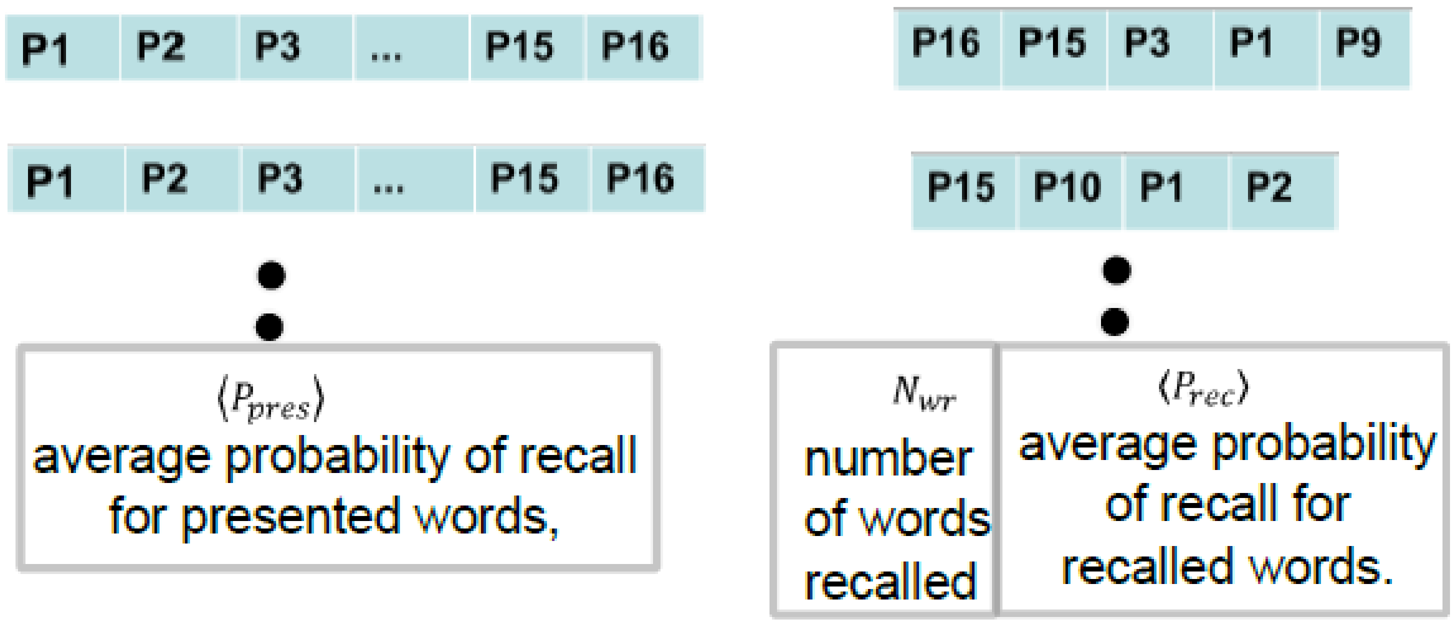
$$L = 16$$

‘Easy’ vs ‘difficult’ words

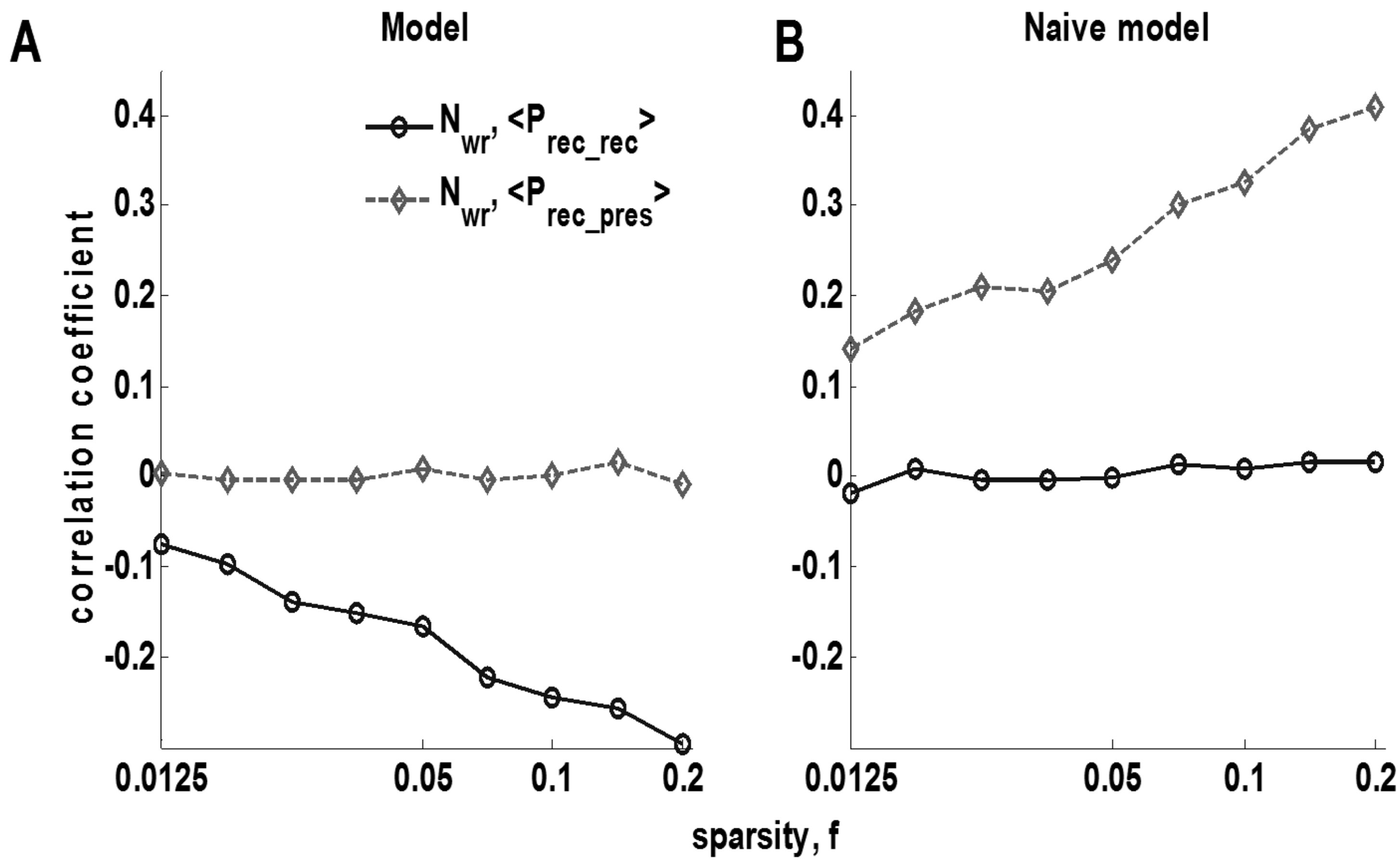


Katkov et al, 2014

More subtle recall statistics



More subtle recall statistics



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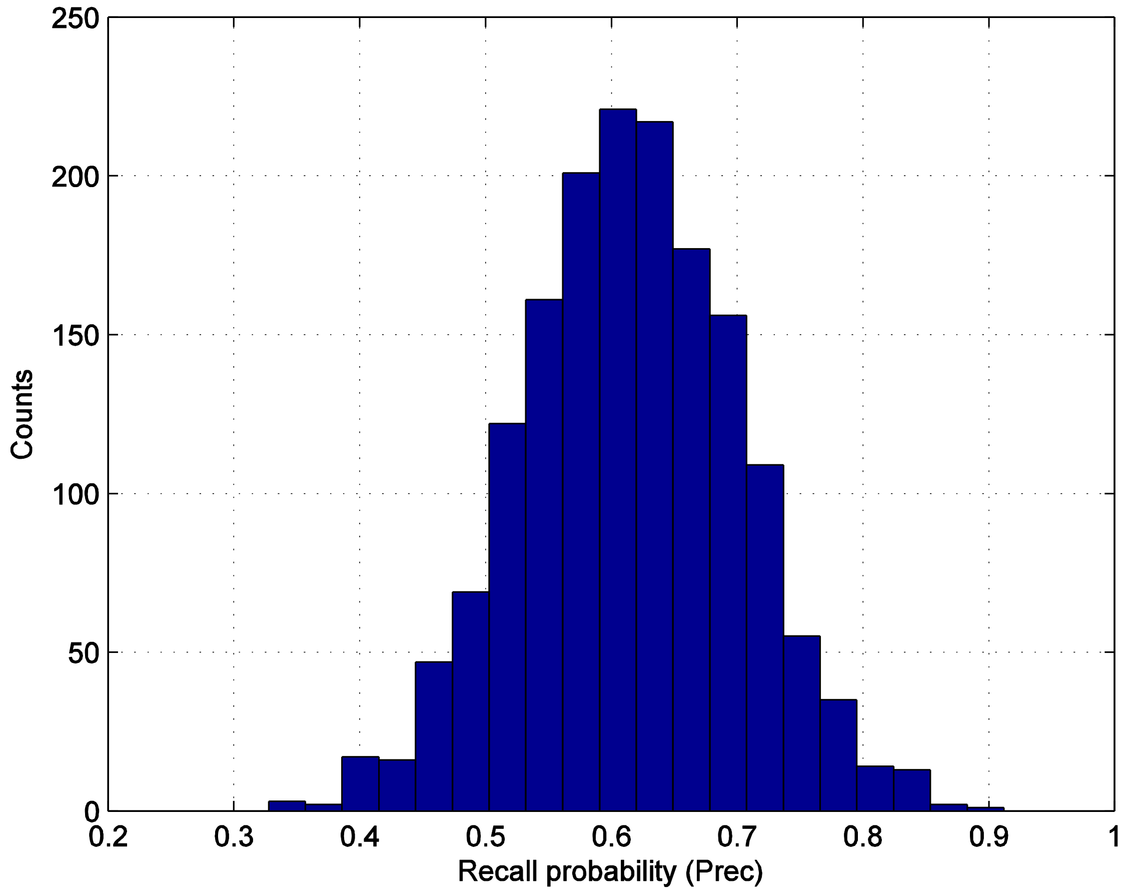
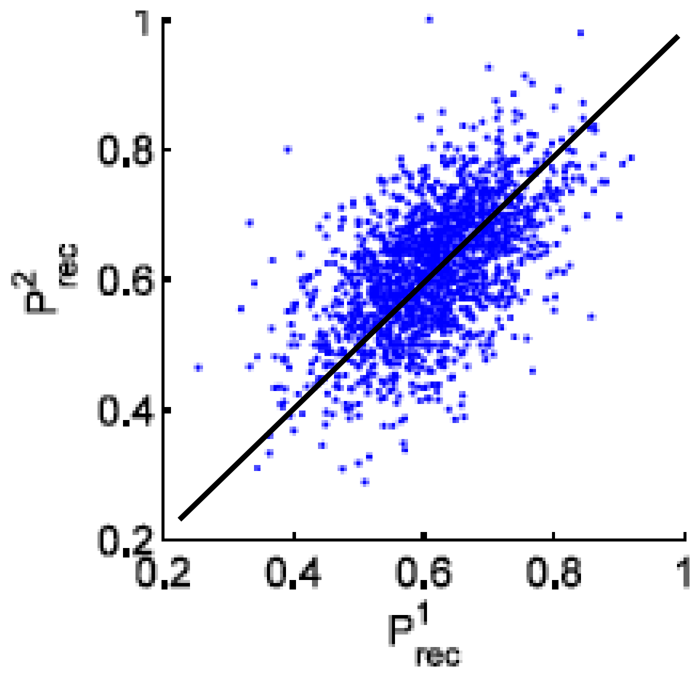
Model predictions

- Easy vs difficult words
- Nontrivial interactions between recall of easy vs difficult words (‘shielding’)

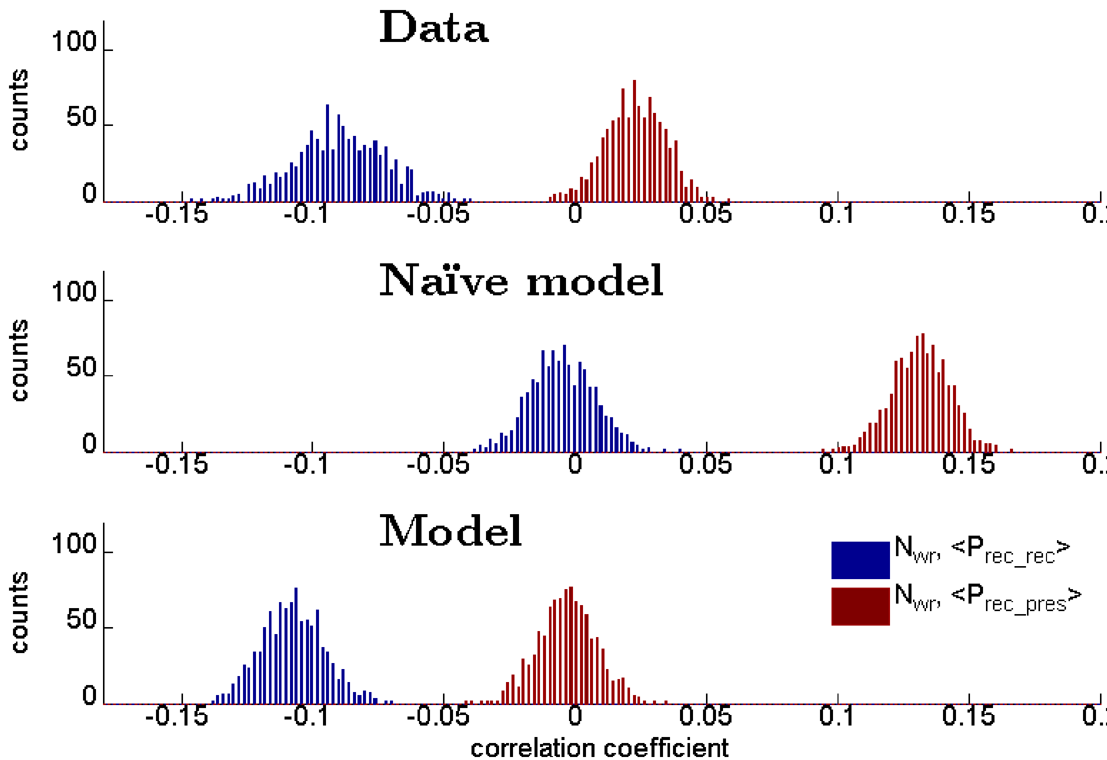
Distribution of recall probabilities over a pool of 1638 words (141 subjects, 112 trials/subject, L=16)



Easy vs difficult words



Recall statistics: data vs model



Katkov et al, 2014

Summary

- Randomness of long-term memory representations results in repeated recall of same items and hence limits the recall capacity.
- Power-law scaling of retrieval capacity emerges from random distribution of transitions between different items.
- Recall capacity can be improved by applying recall strategies based on temporal presentation order.